TWO-MODE SQUEEZED STATES IN HIGH ENERGY PHYSICS: KEY TO HADRON MODIFICATION IN DENSE MATTER ?

M. ASAKAWA
Department of Physics, School of Science, Nagoya University, Nagoya, 464 - 01, Japan

T. CSÖRGŐ
MTA KFKI RMKI, H - 1525 Budapest 114, POB. 49, Hungary
Department of Physics, Columbia University, 538 W 120-th Street, New York, NY 10027, USA

We discuss the possibility to observe hadron modification in hot and dense matter via the correlation of identical particles. We find that a modification of hadronic masses in medium leads to two-mode squeezing which signals itself in back-to-back correlations of hadrons. Although this effect leads to a signal of a shift in ϕ-mass at RHIC, larger signal is expected for the mass shift of kaons or pions.

1 Introduction

The Hanbury-Brown Twiss (HBT) effect has been widely measured in heavy ion collisions. It has been expected that the effect will give some clue to the size of the system at freeze-out. Another interesting topic in heavy ion physics is the possibility of hadron modification in medium. So far these two have been considered as two different aspects of heavy ion physics. The HBT effect is concerned with freeze-out where interaction disappears, whereas the hadron modification is caused by interaction. However, this picture is purely classical. Since the HBT effect is of quantum nature, we need quantum mechanical consideration.

In relativistic heavy ion collisions, freeze-out looks rather prompt. The distribution of final state hadrons is, in most of the cases, almost exponential [1] and this suggests that the system is almost thermalized up to some time and then breaks up suddenly [2]. Motivated by this, we have modeled [3,4] freeze-out as follows. The system remains thermalized until freeze-out. Hadrons are modified due to interaction and their masses are shifted. As a result, it is dressed pseudo-particles that are thermalized. Then, freeze-out is assumed to take place suddenly. This scenario was investigated for small modification of pion and kaon masses in ref. [4]. Here we summarize the correct theoretical results, following the ideas of ref. [4], and will apply the results to the correlation signal of mass modification of ϕ mesons.

2 Calculation of Correlation Functions

In this paper, we use the following scalar theory and we ignore the isospin structure, because it is not essential in our argument. Throughout this paper, we adopt the mean field approximation for simplicity and for clarity. In addition, we do not take into account the finiteness of the system. Calculations for finite systems will be presented elsewhere.

The theory in the vacuum is given by the following free Lagrangian $\mathcal{L}_0$:

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) - \frac{1}{2}m_0^2\phi^2(x), \tag{1}$$

where m_0 is the vacuum mass of the scalar field $\phi(x)$. After standard canonical quantization and normal ordering, we get the well-known diagonalized Hamiltonian, $H_0 = \int \omega_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} d^3\mathbf{k}$, where $\omega_{\mathbf{k}} = \sqrt{m_0^2 + \mathbf{k}^2}$. When the temperature and/or chemical density is non-vanishing and the mass of the ϕ field is shifted, within the mean field approximation, this is expressed by the following Lagrangian in medium $\mathcal{L}_M$:

$$\mathcal{L}_M = \frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) - \frac{1}{2}m_*^2\phi^2(x). \tag{2}$$

The mass shift δM is given by $\delta M = m_* - m_0$. Let us define the quanta that diagonalize the Lagrangian (2) as $b_{\mathbf{k}}$. The point here is that the b-quanta are, in general, different from the a-quanta which are the fundamental excitations in the vacuum. In other words, the a-operators do not diagonalize the Hamiltonian in medium H_M. By normal ordering of H_M, we obtain

$$\begin{aligned} H_M &= H_0 + H_1, \\ H_1 &= \frac{m_*^2 - m_0^2}{4}\int \frac{1}{\omega_{\mathbf{k}}}[a_{\mathbf{k}}a_{-\mathbf{k}} + 2a_{-\mathbf{k}}^\dagger a_{-\mathbf{k}} + a_{\mathbf{k}}^\dagger a_{-\mathbf{k}}^\dagger]\, d^3\mathbf{k}. \end{aligned} \tag{3}$$

Therefore, in medium, mode $\mathbf{k}$ and mode $-\mathbf{k}$ of the a-quanta are mixed. This Hamiltonian (3) can be exactly diagonalized with the following Bogoliubov transformation:

$$\begin{aligned} a_{\mathbf{k}}^\dagger &= \cosh r_{\mathbf{k}}\, b_{\mathbf{k}}^\dagger + \sinh r_{\mathbf{k}}\, b_{-\mathbf{k}}, \quad & a_{-\mathbf{k}}^\dagger &= \cosh r_{-\mathbf{k}}\, b_{-\mathbf{k}}^\dagger + \sinh r_{-\mathbf{k}}\, b_{\mathbf{k}}, \\ a_{\mathbf{k}} &= \sinh r_{\mathbf{k}}\, b_{-\mathbf{k}}^\dagger + \cosh r_{\mathbf{k}}\, b_{\mathbf{k}}, \quad & a_{-\mathbf{k}} &= \sinh r_{-\mathbf{k}}\, b_{\mathbf{k}}^\dagger + \cosh r_{-\mathbf{k}}\, b_{-\mathbf{k}}, \end{aligned} \tag{4}$$

where $r_{\mathbf{k}} = 0.5\log\left(\omega_{\mathbf{k}}/\Omega_{\mathbf{k}}\right)$ and $\Omega_{\mathbf{k}} = \sqrt{m_*^2 + \mathbf{k}^2}$. This gives the exact relationship between the quanta in the vacuum (a-quanta) and the quanta in medium (b-quanta) in this theory. With the b-operators, the in-medium Hamiltonian (3) is diagonalized as $H_M = \int \Omega_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} d^3\mathbf{k}$. This is almost trivial, since

the mass of the ϕ field is shifted to m_* in medium. However, the important point here is that it is the b-quanta that are thermalized in medium, since the b-operators diagonalize the in-medium Hamiltonian. On the other hand, the b-quanta are not observed experimentally. It is the a-quanta that are observed. Therefore, in calculating final state observables, we have to evaluate the expectation value of operators defined in terms of a and $a^\dagger$ operators, $\mathcal{O}(a, a^\dagger)$ with the density matrix defined in the b-basis, ρ_b, i.e., $\langle \mathcal{O}(a, a^\dagger) \rangle = \mathrm{Tr}\, \rho_b \mathcal{O}(a, a^\dagger)$. The calculation of the one and two-particle distribution functions which are needed to obtain the two-particle correlation is straightforward, but the Glauber – Sudarshan representation of the thermal density matrix [5],

$$\rho_b = \prod_{\mathbf{k}} \int \frac{d^2\beta_{\mathbf{k}}}{\pi} P(\beta_{\mathbf{k}}) |\beta_{\mathbf{k}}\rangle\langle\beta_{\mathbf{k}}|, \tag{5}$$

is useful [3]. Here $|\beta_{\mathbf{k}}\rangle$ is a coherent state satisfying $b_{\mathbf{k}}|\beta_{\mathbf{k}}\rangle = \beta_{\mathbf{k}}|\beta_{\mathbf{k}}\rangle$ and

$$P(\beta_{\mathbf{k}}) = \frac{1}{n_{\mathbf{k}}} \exp\left(-\frac{|\beta_{\mathbf{k}}|^2}{n_{\mathbf{k}}} \right), \tag{6}$$

where $n_{\mathbf{k}}$ is a Bose–Einstein distribution function with mass m_*:

$$n_{\mathbf{k}} = \frac{1}{\exp(\sqrt{m_*^2 + \mathbf{k}^2}/T) - 1}. \tag{7}$$

We obtain the following measurable one-particle distribution in the final state:

$$N_1(\mathbf{k}) \; = \; \langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \rangle = |c_{\mathbf{k}}|^2 \, n_{\mathbf{k}} + |s_{-\mathbf{k}}|^2 n_{-\mathbf{k}} + |s_{-\mathbf{k}}|^2, \tag{8}$$

where $c_{\mathbf{k}} = \cosh r_{\mathbf{k}}$, $s_{\mathbf{k}} = \sinh r_{\mathbf{k}}$ (similar expressions hold for $-\mathbf{k}$). If we assume the uniformity of the system, i.e., $n_{\mathbf{k}} = n_{-\mathbf{k}}$ and $r_{\mathbf{k}} = r_{-\mathbf{k}}$, the spectrum of eq. (8) will become the same as the spectrum obtained for one-mode squeezed states [4]. Since we have ignored the finite size effect, the two-particle distribution function $\langle a^\dagger_{\mathbf{k}} a^\dagger_{\mathbf{k}'} a_{\mathbf{k}} a_{\mathbf{k}'} \rangle$ takes a trivial value $\langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \rangle \langle a^\dagger_{\mathbf{k}'} a_{\mathbf{k}'} \rangle$ unless $\mathbf{k} = \pm\mathbf{k}'$. We get the non-trivial two-particle correlation functions $C_2(\mathbf{k}, \pm\mathbf{k})$ as follows:

$$\begin{aligned} C_2(\mathbf{k}, \mathbf{k}) \;\; &= \;\; 2(!), \\ C_2(\mathbf{k}, -\mathbf{k}) \;\; &= \;\; 1 + \frac{|c^*_{\mathbf{k}} s_{\mathbf{k}} n_{\mathbf{k}} + c^*_{-\mathbf{k}} s_{-\mathbf{k}} n_{-\mathbf{k}} + c^*_{-\mathbf{k}} s_{-\mathbf{k}}|^2}{N_1(\mathbf{k})\, N_1(-\mathbf{k})} \neq 1 \; . \end{aligned} \tag{9}$$

In all other cases, the two-particle correlation function is 1. This result implies two important issues, (i) the intercept of the correlation function $C_2(\mathbf{k}, \mathbf{k})$

remains the canonical value of 2, even if quanta in medium are different from those in the vacuum, (ii) back-to-back correlation ($C_2(\mathbf{k}, -\mathbf{k}) \neq 1$) is generated by hadron modification. This is caused by the mixing of $\mathbf{k}$ and $-\mathbf{k}$ modes due to the mean field effect.

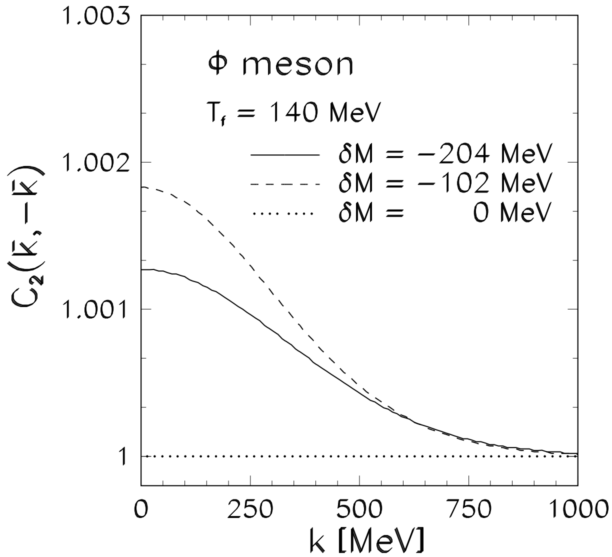

Fig. 1. Large mass-shift of ϕ results in a small back-to-back correlation. Solid line stands for a 10 %, dashed line for a 20 % decrease in m_ϕ. The freeze-out temperature was assumed to be $T = 140$ MeV. Similar effect for pions leads to bigger signal.

Thus the correlation properties of two-mode squeezed states are essentially different from those properties of the one-mode squeezed states, that were invoked in ref. [3]. In our case, the mean field carries no momentum. Due to momentum conservation, only $\mathbf{k}$ and $-\mathbf{k}$ modes get additional correlation. In contrast, when one mode squeezing is assumed, $C_2(\mathbf{k}, -\mathbf{k}) = 1$ and one can show that the intercept of the two-particle Bose-Einstein correlation function may take up any non-negative value: $0 < C_2(\mathbf{k}, \mathbf{k}) < \infty$. Although the importance of the back-to-back correlation functions was well realized in ref [4], the formula for the back-to-back correlation function was incorrectly given by eq. (18) of ref. [4]. The correct expression for the back-to-back correlation function is presented by our eq. (9).

Since back-to-back correlations of ϕ mesons may be observable by the PHENIX detector at RHIC [6], (the Relativistic Heavy Ion Collider which shall be operational by 1999 colliding $Au + Au$ at $\sqrt{s} = 40$ TeV) we apply here

our model to the case of a 10 - 20 % mass-shift of ϕ-mesons on Fig. 1. The back-to-back correlation function of ϕ mesons has a worse signal-to-noise ratio, than that of kaons, which is due to the heavier mass of ϕ ($m_\phi = 1020$ MeV vs. $m_K = 494$ MeV). Fig. 1. indicates that a 10 - 20 % shift in the mass of ϕ mesons results in a small signal in the back-to-back correlation function.

3 Summary

A new method has been found to test the medium modification of bosons, utilizing their quantum correlations at large momentum difference. The effect follows from basic principles of statistical physics and canonical quantization.

Back-to-back correlations are not contaminated by resonance decays at larger momentuma of the particles, in contrast to the Bose–Einstein correlation functions at small relative momenta. For a locally thermalized expanding source, however, back-to-back correlations will appear in the rest frame of each fluid element. Thus, for such systems a more realistic estimate is necessary to learn more about the magnitude of the correlations of ϕ or other mesons at large relative momenta. Contrary to previous expectations [3,4] the effect is the biggest for the lightest particles (photons, pions or kaons) and the back-to-back correlations vanish for very large values of particle momenta. In the present study we have neglected finite size effects, which will make the correlation function vary smoothly around both $\mathbf{k_1} = \mathbf{k_2}$ and $\mathbf{k_1} = -\mathbf{k_2}$.

Acknowledgment

We would like to thank M. Gyulassy and J. Knoll for enlightening discussions. T. Csörgő is grateful to Györgyi and M. Gyulassy for kind hospitality at Columbia University. This research has been supported by the US - Hungarian Joint Fund under contract MAKA 378/1993, by the Hungarian NSF under contract OTKA - F 4019, T024094 and W01015107 and by an Advanced Research Award of the Fulbright Foundation, grant document 20926/1996.

1. Proceedings of *Quark Matter '95*, *Nucl. Phys.* **A590** (1995) Nos. 1, 2.
2. T. Csörgő and L. P. Csernai, *Phys. Lett.* **B333** (1994) 494.
3. I. Andreev and R. M. Weiner, *Phys. Lett.* **B373** (1996) 159.
4. M. Asakawa and T. Csörgő, hep-ph/9612331,
   Heavy Ion Physics **4** (1996) 233-240
5. R. J. Glauber, *Phys. Rev.* **131** (1963) 2766;
   E.C.G. Sudarshan, *Phys. Rev. Lett.* **10** (1963) 277.
6. W. A. Zajc and B. Lörstad, private communication.